Exam II /75

Name: KEY

1) Roy goes out to the bar deciding that he will approach ten girls and ask for their phone number. Historical data tells us that Roy is successful in getting a girl's number 0.5% of the time. Roy is interested in whether or not he'll get any numbers that night.

a. What type of distribution would X = the number of phone numbers Roy gets follow?

Binomial

b. What are the parameters of the model?

$n = 10$

$p = .5\% = .005$

c. What is the probability that Roy gets exactly one phone number? Do this in R.

$$P(X=1) = \binom{10}{1} .005^1 (1-.005)^9$$
$$= 10 \cdot .005 \cdot .995^9$$
$$= .0477944789$$

d. What is the probability that Roy doesn't get any phone numbers? Do this by hand and check it in R.

$$P(X=0) = \binom{10}{0} .005^0 (1-.005)^{10}$$
$$= 1 \cdot 1 \cdot .995^{10}$$
$$= .95111013$$

2) http://www.cdc.gov/nchs/data/series/sr_11/sr11_252.pdf

Roy, the same night he was going out to ask girls for the phone numbers, looked in the mirror and thought – "I feel fat." He told me this and I decided I would use statistics to make him feel better. The weight, in pounds, of Americans between the ages of 20 and 29 is follows a symmetric, mound shaped distributed with a mean of 183.9 pounds and a standard deviation of 55.91269.

a. What type of distribution would X = Roy's weight follow?

Normal

b. What are the parameters of the model?

$\mu = 183.9$

$\sigma = 55.91269$

c. If Roy weighs 155 pounds, what proportion of Americans does Roy weigh less than? Use R here.

$$Z = \frac{155 - 183.9}{55.91269} = -.5168773$$

$$P(X < 155) = P(Z < -.5168773$$

$$= .3026209$$

d. If Roy weighs 155 pounds, how much weight does Roy have to lose or gain to be at the 50[th] percentile? Use the Z-table here.

50th percentile $\rightarrow Z = 0 \rightarrow X = \mu = 183.9$

$$155 - 183.9 = -28.9$$

He needs to gain 28.9 pounds.

3) http://www.cbsnews.com/news/state-universities-increasingly-recruiting-rich-students/

Suppose 29.1 percent (9,306 of 31,980) of students at The University of South Carolina receive merit based scholarships. Find the probability that more than a third of a random sample of 72 receive merit based scholarship.

a) What is the mean of the sampling distribution of the sample proportion?

$$\mu_{\hat{p}} = p = .291$$

b) What is the standard error of the sampling distribution of the sample proportion?

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.291(1-.291)}{72}} = .05353075$$

c) Is it safe to assume that the sampling distribution of the sample proportion is bell shaped?

$$np = 72 \cdot (.291) = 20.952$$
$$n(1-p) = 72 \cdot (1-.291) = 51.048$$
Yes!

d) What is the probability that more than a third of the random sample of 72 receive merit based scholarship, use the Z-table.

$$P\left(\hat{p} > \tfrac{1}{3}\right) = P\left(Z > \frac{\frac{1}{3} - .291}{.05353075}\right)$$
$$= P(Z > .7908227)$$
$$= 1 - P(Z < .7908227)$$
$$= .2145237$$

e) Double check this in R:

pnorm(1/3, .291, .05353075)

4) http://www.wltx.com/story/sports/ncaa/usc-gamecocks/2015/06/03/wltx-gamecocks-gpa/28413937/
In the spring semester of 2015, student athletes at the University of South Carolina posed an average GPA of 3.256. Suppose the population standard deviation is .5; find the probability that a random sample of 25 student athletes has an average GPA over 3.5.

a) What is the mean of the sampling distribution of the sample mean?

$$\mu_{\bar{x}} = \mu_x = 3.256$$

b) What is the standard error of the sampling distribution of the sample mean?

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.5}{\sqrt{25}} = .1$$

c) Is it safe to assume that the sampling distribution of the sample mean is bell shaped?

No, $n < 30$ and it is not explicitly stated that the GPA follows the normal dist. We can view the boxplot / histogram & proceed w/ caution.

d) What is the probability that a random sample of 25 student athletes has an average GPA over 3.5?

$$P(\bar{x} > 3.5) = P\left(z > \frac{3.5 - 3.256}{.1}\right)$$

$$= 1 - P(z \leq 2.44)$$

$$= 1 - .9926564$$

$$= .007343631$$

e) Double check this in R:

1 pnorm(3.5, 3.256, .1)

5) http://www.cnn.com/2015/06/01/politics/tsa-failed-undercover-airport-screening-tests/
67 out of 70 tests conducted by the Department of Homeland Security resulted in banned items getting through the screening process. Find an interval estimate for the population.

a) What is the target parameter?

Population proportion p

b) What is the point estimate for the target parameter?

$\hat{p} = 67/70 = .9571429$

c) What is the confidence coefficient for a 95% confidence interval?

$z = 1.96$

d) What is the margin of error?

$$MOE = 1.96\left(\sqrt{\frac{.9571429(1-.9571429)}{70}}\right)$$

$$= .04744679$$

e) Using parts a through d, find a 95% confidence interval for the target parameter and interpret the interval in the context of the problem – include any assumptions. Do this by hand.

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$.9571429 \pm .04744679$$

$$(.9096961, 1.00459)$$

$n\hat{p} = 60 \cdot .9571429 = 57.42857 > 15$

$n(1-\hat{p}) = 60 \cdot (1-.9571429) = 2.571426 < 15$

proceed w/ caution!

(Wilson's correction would be a good idea here)

f) Check part e using R.

6) The survey sent out in the beginning of the semester, filled out by 12 students, showed that self-attractiveness rating followed a normal distribution with mean 7.3333 and a standard deviation of 1.4355. Find an interval estimate for the population.

a) What is the target parameter?

The population mean μ

b) What is the point estimate for the target parameter?

$\bar{X} = 7.\overline{33}$

c) What is the confidence coefficient for a 95% confidence interval?

$t_{1-\frac{.05}{2},\,12-1} = qt(.975, 11) = 2.200985$

d) What is the margin of error?

$t_{1-\frac{.05}{2},\,12-1}\left(\frac{1.4355}{\sqrt{12}}\right) = .9120727$

e) Using parts a through d, find a 95% confidence interval for the target parameter and interpret the interval in the context of the problem – include any assumptions. Do this by hand.

$7.\overline{33} \pm .9120727$

$(6.42161, 8.245406)$

- $n < 12$
- X is not explictly stated as normal
- We can look at a histogram or boxplot

* Proceed w/ caution *

f) Check part e using R.